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A generalized correlation for equilibrium of forces in liquid–solid fluidized beds

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Abstract

Following a critical publication review on the expansion of fluidized bed with the fluid velocity, the well-known Richardson–Zaki equation is applied to develop a more accurate relationship linking the apparent drag force F_d, the effective gravitational force F_g and the voidage of fluidized beds under intermediate regime. A correcting constant a in the relationship is a function of Archimedes number which may provide a degree of flexibility for different systems covering a wide range of variables. With addition of the correlation developed in this work, equivalence correlations between equilibrium forces and the Richardson–Zaki equation can be found for a whole regime.

Keywords: Apparent drag force; Effective gravitational force; Expansion; Fluidization; Voidage

1. Introduction

Fluidization is very important from an industrial point of view. The processes of liquid–solid fluidization are widely utilized in the fields of hydrometallurgy, food technology, biochemical processing, water treatment, etc. The operations involved may include crystallization, ion exchange, adsorption, enzyme catalysis and cell culturing, etc. Scientific fundamental research concerns the hydrodynamic structure of liquid-particles, the equilibrium forces for fluid-particle interactions and heat or mass transfer properties in fluidized beds. Taking into account the physical parameters of a liquid–solid system, the expansion characteristic of solid particles in the bed is a function of superficial liquid velocity. A quantitative relationship linking the expansion and these parameters is necessary for a fundamental understanding of fluidization behavior and subsequent application.

Following a critical review of previous publications [1–25] on the interaction between particles and fluid, it is recognized that the forces on a single particle in a fluid can be clearly distinguished and have been exhaustively correlated for a wide range of *Re*. However, when the particle is in suspension, statements dealing with the interaction between any one particle and its surroundings become increasingly difficult due to the complex structure of fluid flow, random interactions among the particles and fluid-particles. Not surprisingly, the prediction of the published correlations for the interaction of liquid-particles in fluidized bed shows wide differences. Therefore, more precise analysis of liquid-particle interaction is required in order to have a more accurate prediction of the fluidized bed expansion.

After a short analysis of the force equilibrium correlations for a particle in suspension, an inherent link between the force equilibrium correlation and the Richardson–Zaki equation has been found at two extreme conditions. However, at the intermediate region, no correlation is equivalent to the Richardson–Zaki equation. In this work an attempt has been made to employ the Richardson–Zaki equation for the development of force equilibrium correlation. A new correlation is proposed that provides more reliable calculation for the interaction of liquid-particles in suspension.

2. Steady-state expansion characteristics

The expansion properties of fluidized beds have been extensively studied. Experimental data demonstrate that the voidage–velocity relationship is independent of the total mass of solid particles in a liquid fluidized bed. The different relationships between the superficial liquid velocity u, the terminal velocity u_t, and the voidage, ε, have been developed. Most of works concerning the expansion of fluidized beds have been summarized by Couderc [26]. The first important work in this direction was made by Richardson and

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Nomenclature

- a constant in Eq. (17)
- b constant in Eq. (17)
- Ar Archimedes number $((d_p^3 g(\rho_p - \rho)\rho)/\mu^2)$
- C_D drag coefficient for a single particle
- d_p particle diameter (m)
- F_d apparent drag force (kg m s^{-2})
- F_g effective gravitational force (kg m s^{-2})
- g gravitational acceleration constant (m s^{-2})
- n exponent in Richardson–Zaki equation
- Re particle Reynolds number, $((d_p u \rho)/\mu)$
- Re_t terminal Reynolds number, $((d_p u_t \rho)/\mu)$
- u superficial velocity based on an empty column (m s^{-1})
- u_t terminal settling velocity of a single particle (m s^{-1})
- V_p particle volume (m^3)

Greek symbols

- α correcting factor in Eq. (17)
- ε voidage of fluidized bed
- ρ fluid density (kg m^{-3})
- ρ_p apparent particle density (kg m^{-3})
- μ liquid viscosity (mPa s)

Zaki [27] where the following equation for the fluidized bed expansion was proposed:

$$\frac{u}{u_t} = \varepsilon^n \tag{1}$$

The exponent n is a function of terminal particle Re (in the absence of wall effects). Based on bed expansion data, four separate equations were recommended and each one spanned a limited range of terminal particle Re:

$$\begin{aligned} n &= 4.65 && Re_t < 0.2 \\ n &= 4.45 Re_t^{-0.03} && 0.2 < Re_t < 1 \\ n &= 4.45 Re_t^{-0.1} && 1 < Re_t < 500 \\ n &= 2.39 && 500 < Re_t \end{aligned} \tag{2}$$

The validity of Richardson–Zaki equation has been confirmed overwhelmingly with the experimental data and it provides an excellent account of expansion characteristics of mono-size spherical particles of the same material in fluidized beds. Therefore, in this work, the Richardson–Zaki equation is used to correlate the fundamental forces (gravity, buoyancy and drag) between the particle and the liquid in fluidized suspension.

3. Analysis of force equilibrium of particle in suspension

For a bed of solid particles to be in the fluidized state, where each particle is completely supported by the liquid or the suspension, the total interaction forces exerted on it must match its weight. In the case of a single particle in an infinite expansion state ($\varepsilon = 1$), the total interaction can be conveniently divided into the drag and the buoyant forces:

$$V_p \rho_p g = \text{drag} + \text{buoyancy} \tag{3}$$

or

$$\tfrac{1}{6}(\pi d_p^3)\rho_p g = C_D \tfrac{1}{4}(\pi d_p^2)\tfrac{1}{2}(u^2 \rho) + \tfrac{1}{6}(\pi d_p^3)\rho g$$

However, how to distinguish the drag and the buoyant forces acting on a single particle in a fluidized bed is still a difficult problem. The difficulties stem mainly from the complex structure of the fluid flow and the intensity of random interactions among the particles and fluid-particle [28]. Experimental resolution is impossible as only total fluid-particle interaction forces can be directly measured in multi-particle systems.

If one defines that the drag force acting on an isolated single particle at liquid apparent velocity $[C_D(\pi d_p^2/4)(u^2\rho/2)]$ as apparent drag force F_d, and the weight of a solid particle in liquid $[(\pi d_p^3 g/6)(\rho_p - \rho)]$ as effective gravitational force F_g, the ratio of the drag force (F_d) of fluid acting on an isolated single particle to effective gravitational force (F_g) is equal to 1 in an infinite expansion state. The ratio of F_g/F_d is a function of voidage in multi-particle liquid fluidized bed and has the following form:

$$C_D \tfrac{1}{4}(\pi d_p^2)\tfrac{1}{2}(u^2\rho) = \tfrac{1}{6}(\pi d_p^3 g)(\rho_p - \rho) f(\varepsilon) \tag{4}$$

i.e. $F_d = F_g f(\varepsilon)$.

Lewis et al. [1], Wen and Yu [5] and Kmiec [8] obtained similar results, showing that:

$$C_D \tfrac{1}{4}(\pi d_p^2)\tfrac{1}{2}(u^2\rho) = \tfrac{1}{6}(\pi d_p^3 g)(\rho_p - \rho)\varepsilon^n \tag{5}$$

This equation can be rewritten in a dimensionless form as

$$\frac{3}{4} C_D \frac{Re^2}{Ar} = \varepsilon^n \tag{6}$$

where $n = 4.65$ for Lewis et al., $n = 4.7$ for Wen and Yu, $n = 4.78$ for Kmiec.

The similar correlation form as Eq. (5) or (6) may be obtained by transforming the Richardson–Zaki equation with the aid of Stokes' law and Newton equation.

In the region of very low Re, the Richardson–Zaki equation can be transformed using the Stokes' law:

$$Ar = 18 Re_t, \qquad Ar < 3.6 \quad \text{and} \quad Re_t < 0.2 \tag{7}$$

Substitution of this equation into the Richardson–Zaki equation leads to

$$\frac{Re}{Re_t} = \frac{18}{Re}\frac{Re^2}{Ar} = \varepsilon^{4.65}, \qquad Re_t < 0.2 \tag{8}$$

where

$$\frac{18}{Re} = \frac{3}{4} \times \frac{24}{Re} = \frac{3}{4} C_D$$

Hence Eq. (8) becomes

$$\frac{3}{4}C_D\frac{Re^2}{Ar} = \varepsilon^{4.65}, \quad Re_t < 0.2 \tag{9}$$

This equation can be rearranged as:

$$C_D\frac{1}{4}(\pi d_p^2)\frac{1}{2}(u^2\rho) = \frac{1}{6}(\pi d_p^3 g)(\rho_p - \rho)\varepsilon^{4.65}, \quad Re_t < 0.2 \tag{10}$$

For Newton's region [7], the same transformation processes can be made as following:

$$Ar = 0.33Re_t^2, \quad Ar > 10^5, 500 < Re_t < 2\times10^5 \tag{11}$$

According to the Richardson–Zaki equation, one has

$$\frac{Re}{Re_t} = \sqrt{\frac{0.33}{Ar}}Re = \varepsilon^{2.39}, \quad Re_t > 500 \tag{12}$$

This equation is equivalent to

$$0.33\frac{Re^2}{Ar} = \varepsilon^{4.78} \tag{13}$$

The drag coefficient C_D for an isolated sphere in this region is equal to 0.44 [29], so one obtains

$$\frac{3}{4}C_D\frac{Re^2}{Ar} = \varepsilon^{4.78}, \quad Re_t > 500 \tag{14}$$

This equation can also be rearranged as:

$$C_D\frac{1}{4}(\pi d_p^2)\frac{1}{2}(u^2\rho) = \frac{1}{6}(\pi d_p^3 g)(\rho_p - \rho)\varepsilon^{4.78}, \quad Re_t > 500 \tag{15}$$

The analysis of equilibrium forces indicates that there are some inherent links between the force balance correlation and the Richardson–Zaki equation at two extreme conditions. However, at intermediate region, this equation cannot be directly transformed to the force equilibrium form and no equivalent correlation is available.

The exponent of voidage in Eq. (5) should be situated between 4.65 and 4.78 at the intermediate region. This exponent is not a fixed value as suggested by Wen and Yu [5] and Kmiec [8], but rather is dependent on the system properties and expansion state. It should be noted that a small difference in the exponent will result in a large deviation for the balance of effective gravitational force F_g and apparent drag force F_d. When the solid particles are at incipient fluidization state ($\varepsilon \approx 0.4$), the ratio of the effective gravitational force and the apparent drag force is about 80 calculated with $n = 4.78$ and 74 with $n = 4.7$. This implies that the ratio of F_g/F_d is very sensitive to the exponent value. Therefore, the assumptions of $f(\varepsilon)$ in Eq. (4) as the form ε^n and taking n as a constant are inappropriate. An accurate function relationship is required to obtain a suitable force equilibrium correlation for different systems.

4. Generalization of force equilibrium correlations

Due to the fact that the Richardson–Zaki equation has been confirmed overwhelmingly with the available data, it is a nearly universally adopted velocity–voidage relationship in fluidized beds. In addition to its direct application, the Richardson–Zaki equation was commonly employed in theoretical studies [30] and some successful attempts have been made [31].

The following work is to revise Eq. (15) on the basis of the Richardson–Zaki equation at the intermediate region. Thus the equilibrium force correlations will correspond to the Richardson–Zaki equation over a whole range of *Re*.

By comparing the ratio of right hand side (RHS) and left hand side (LHS) of Eq. (15) with the values of voidage calculated by the Richardson–Zaki equation, one finds that there is a significant difference at the low range of voidage and also that the deviation degree is not the same for different systems (see Fig. 1). Therefore, the crucial problem lies in how to correct these deviations.

An attempt has been made to correlate the exponent n in Eq. (5) as a function of Ar or Re_t [32], no simple function form was found. Further fitting results indicate that Eq. (15) can be improved by adding a correcting factor α:

$$\alpha C_D\frac{1}{4}(\pi d_p^2)\frac{1}{2}(u^2\rho) = \frac{1}{6}(\pi d_p^3 g)(\rho_p - \rho)\varepsilon^{4.78} \tag{16}$$

i.e.

$$\alpha\varepsilon^{-4.78} = \frac{F_g}{F_d}$$

The correcting factor α in Eq. (16) may be chosen to match more closely the data from the Richardson–Zaki equation and provides a degree of flexibility.

As ε is equal to unity, α, also equal to 1 (see Fig. 1), the particle in fluidized bed becomes a single unhindered particle and the drag force F_d from the liquid is equal to the gravity force F_g of the solid particle in the liquid. But as $\varepsilon < 1$, the value of α depends on the properties of system

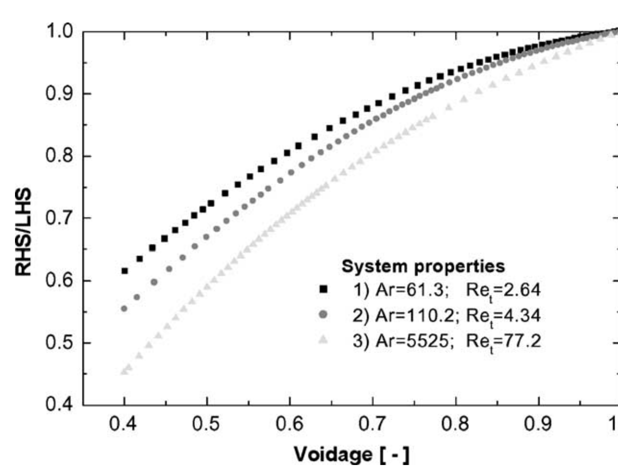

Fig. 1. Comparison of the ratio of RHS and LHS of Eq. (15) using the voidage values from the Richardson–Zaki equation.

Table 1
Physical properties of the solid particles and the liquid[a]

Solid material	d_p (μm)	ρ_p (kg m^{-3})	u_t (m s^{-1})	Re_t[a]	Ar[a]	n
Copper	135	8800	0.42	4.34	110.2	3.84

[a] Properties of water at 10 °C, $\rho = 1000$ kg m^{-3}, $\mu = 1.307$ mPa s.

and is a function of voidage ε. Various forms of the function $\alpha = f(\varepsilon)$ are simulated by using a fitting program and a best form is expressed by:

$$\frac{1}{\alpha} = a + \frac{b}{\varepsilon^2} \tag{17}$$

The values of constants a and b in Eq. (17) can be obtained by fitting with Eq. (16) in which the values of voidage are calculated from the Richardson–Zaki equation.

An example is given to illustrate the fitting processes. The physical properties and relevant parameters in this example are given in Table 1.

The terminal Re in Table 1 is determined with the Schiller and Naumann equation [33], the exponent n is calculated from Eq. (2). With these parameters, the values of constants a and b in Eq. (17) can be obtained as a function of voidage by using table–curve program (see Fig. 2).

The same processes have been made for about 50 systems in the low region of terminal Re ($Re_t < 50$). In these systems, the viscosity and the density of liquid, the diameter and the density of particle are changed, so that the constants in Eq. (17) can be obtained at different terminal Re or different Archimedes numbers. The results are given in Table 2.

Results show that the constants a and b change with the Re_t and Ar. In the low region of terminal Re ($1 < Re_t < 50$), the constants a and b in Eq. (17) are represented with a_1 and b_1. The correlations of a_1 and b_1 with Ar are given in Figs. 3 and 4, respectively, then the following equations are obtained:

$$a_1 = 0.7418 + 0.9670 Ar^{-0.5},$$
$$b_1 = 0.2488 - 0.9239 Ar^{-0.5},$$
$$1 < Re_t < 50,\ 24 < Ar < 3000 \tag{18}$$

For the high region of terminal Re ($50 < Re_t < 500$ and $3000 < Ar < 100{,}000$), the other 35 different systems have been simulated and given in Table 3. In the high region of terminal Re, a and b in Eq. (17) are represented with a_2 and b_2. The data of a_2 and b_2 are plotted in Figs. 5 and 6 as a function of Archimedes number, the fitting results are:

$$a_2 = 0.7880 - 0.00009 Ar^{0.625},$$
$$b_2 = 0.2106 + 0.00011 Ar^{0.625},$$
$$50 < Re_t < 500, \quad 3000 < Ar < 10^5 \tag{19}$$

A summary is made here to obtain a desirable result. After substitution of Eq. (17) into Eq. (16), the correlation between the drag force and the gravity force in fluidized bed can be expressed as follows:

$$C_D \tfrac{1}{4}(\pi d_p^2)\tfrac{1}{2}(u^2\rho) = \tfrac{1}{6}(\pi d_p^3 g)(\rho_p - \rho)(a\varepsilon^{4.78} + b\varepsilon^{2.78}) \tag{20}$$

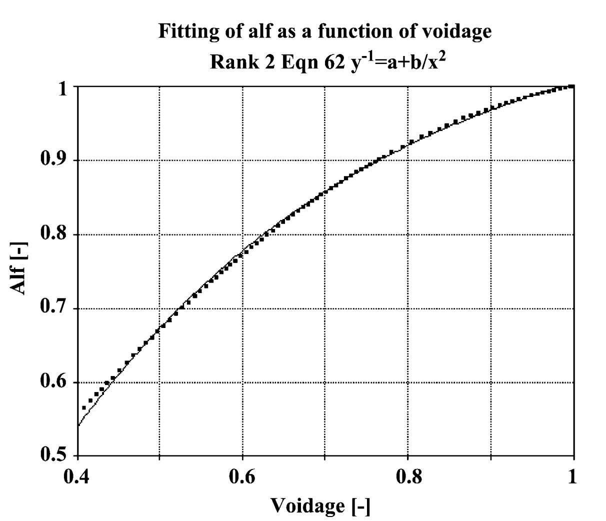

Fig. 2. Illustration of fitting α as a function of voidage with table–curve program.

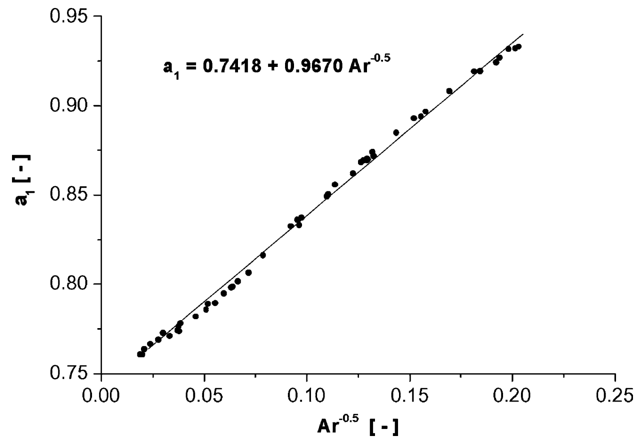

Fig. 3. Dependence of a_1 in Eq. (17) on Archimedes number.

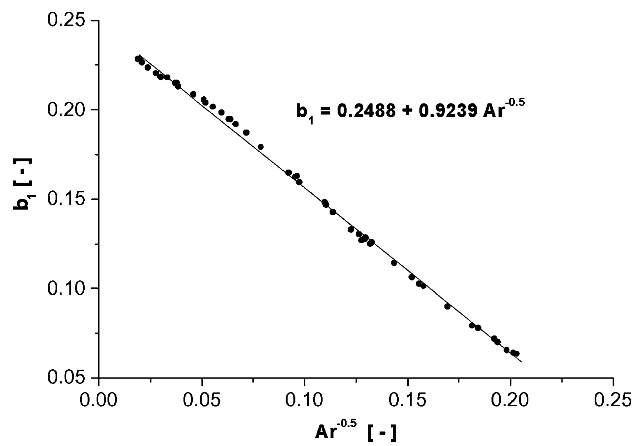

Fig. 4. Dependence of b_1 in Eq. (17) on Archimedes number.

Table 2
Constants in Eq. (17) at low range of terminal Re ($1 < Re_t < 50$, $24 < Ar < 3000$)

No.	Material	d_p (μm)	ρ_p (kg m^{-3})	ρ (kg m^{-3})	μ (mPa s)	Ar	Re_t	n	a	b
1	Amberlite XAD-7	300	1090	998	1.002	24.22	1.15	4.26	0.9329	0.0635
2	PVC	176	1390	998	1.002	24.60	1.17	4.26	0.9321	0.0640
3	Silica gel	180	1350	997	0.890	25.42	1.21	4.25	0.9316	0.0656
4	Amberlite IRA-401	226	1060	988	0.550	26.63	1.26	4.24	0.9266	0.0700
5	Copper	276	8800	997	7.700	27.06	1.28	4.23	0.924	0.0720
6	Amberlite XAD-7	320	1090	998	1.002	29.40	1.38	4.21	0.9193	0.0778
7	Amberlite IRA-401	236	1060	988	0.550	30.32	1.42	4.20	0.919	0.0792
8	Glass beads	135	2450	998	1.002	34.84	1.60	4.17	0.9079	0.0900
9	Glass beads	276	2450	997	2.370	40.09	1.82	4.13	0.8965	0.1013
10	Heavy beads	330	1200	1000	1.307	41.28	1.86	4.12	0.8937	0.1025
11	Heavy beads	280	1200	998	1.002	43.24	1.94	4.11	0.8927	0.1064
12	Amberlite IRA-401	276	1060	988	0.550	48.50	2.15	4.07	0.8846	0.1143
13	Glass beads	310	2450	997	2.370	56.81	2.47	4.03	0.8717	0.1258
14	Amberlite XAD-7	400	1090	998	1.002	57.42	2.49	4.03	0.8739	0.1252
15	Copper	180	8800	997	2.730	59.72	2.58	4.02	0.8694	0.1282
16	Dianon	246	1320	997	0.890	59.37	2.56	4.02	0.8697	0.1284
17	Amberlite IRA-401	296	1060	988	0.550	59.83	2.58	4.02	0.8702	0.1277
18	Glass beads	163	2450	998	1.002	61.34	2.64	4.01	0.8694	0.1270
19	Copper	365	8800	997	7.700	62.59	2.68	4.00	0.8682	0.1304
20	Glass beads	200	2450	1000	1.307	66.62	2.83	3.99	0.8619	0.1329
21	Glass beads	176	2450	998	1.002	77.19	3.22	3.94	0.8556	0.1427
22	Amberlite XAD-7	450	1090	998	1.002	81.75	3.37	3.93	0.8501	0.1468
23	Amberlite IRA-401	330	1060	988	0.550	82.90	3.42	3.92	0.8492	0.1483
24	Glass beads	760	2450	997	7.700	105.22	4.17	3.86	0.8370	0.1596
25	Dianon	300	1320	997	0.890	107.68	4.26	3.85	0.8328	0.1628
26	Copper	135	8800	1000	1.307	110.20	4.34	3.84	0.8358	0.1624
27	Amberlite XAD-7	330	1090	998	0.550	117.45	4.57	3.82	0.8323	0.1647
28	Copper	500	8800	997	7.700	160.90	5.92	3.73	0.8160	0.1792
29	Dianon	365	1320	997	0.890	193.94	6.89	3.68	0.8062	0.1873
30	Copper	560	8800	997	7.700	226.05	7.78	3.64	0.8014	0.1919
31	Heavy beads	330	1200	988	0.550	244.11	8.32	3.61	0.7985	0.1946
32	Copper	290	8800	997	2.370	249.74	8.42	3.61	0.7979	0.1947
33	TiO_2	230	2870	997	0.890	281.39	9.24	3.58	0.7947	0.1983
34	Amberlite IR-120	548	1200	998	1.002	324.16	10.32	3.54	0.7892	0.2016
35	ZrO_2	136	5600	988	0.550	371.71	11.46	3.50	0.7888	0.2038
36	Glass beads	300	2450	998	1.002	382.29	11.71	3.49	0.7856	0.2056
37	Glass beads	216	2450	988	0.550	472.07	13.75	3.44	0.7817	0.2085
38	ZrO_2	166	5600	988	0.550	675.95	17.96	3.34	0.7780	0.2130
39	ZrO_2	250	5600	998	1.002	701.18	18.45	3.33	0.7734	0.2151
40	Amberlite IR-120	710	1200	998	1.002	705.00	18.53	3.33	0.7763	0.2145
41	Dianon	614	1320	998	1.002	726.80	18.93	3.32	0.7741	0.2149
42	Glass beads	400	2450	998	1.002	906.17	22.24	3.27	0.7708	0.2180
43	ZrO_2	276	5600	988	0.550	1112.6	25.78	3.21	0.7726	0.2183
44	Glass beads	450	2450	998	1.002	1290.2	28.64	3.18	0.7688	0.2203
45	Glass beads	500	2450	998	1.002	1769.9	35.75	3.10	0.7663	0.2236
46	Hollow char	776	1500	998	1.002	2287.0	42.69	3.04	0.7634	0.2266
47	ZrO_2	560	2450	998	1.002	2486.5	45.20	3.02	0.7606	0.2284
48	Copper	580	2450	998	1.002	2762.5	48.56	3.00	0.7605	0.2285

Eq. (20) shows that the drag force which a particle received from suspension in fluidized bed is $1/(a\varepsilon^{4.78} + b\varepsilon^{2.78})$-fold of that an unhanded particle received from liquid at the same superficial liquid velocity.

When ε is equal to 1, $(a\varepsilon^{4.78} + b\varepsilon^{2.78})$ is also equal to 1. The drag force from liquid is equal to effective gravity force of an unhanded particle. From this analysis, one finds that $a + b = 1$ and this is valid for a whole range of voidage ($0.4 < \varepsilon < 1.0$). Thus, only one correcting constant is required, Eq. (20) is simplified to

$$C_D \tfrac{1}{4}(\pi d_p^2)\tfrac{1}{2}(u^2\rho) = \tfrac{1}{6}(\pi d_p^3 g)(\rho_p - \rho) \times (a\varepsilon^{4.78} + (1-a)\varepsilon^{2.78}) \qquad (21)$$

$$a = 0.7418 + 0.9670 Ar^{-0.5}, \quad 1 < Re_t < 50, \quad 24 < Ar < 3000 \qquad (22)$$

Table 3
Constants in Eq. (17) at high range of terminal Re ($50 < Re_t < 500$, $3000 < Ar < 100{,}000$)

No.	Material	d_p (μm)	ρ_p (kg m^{-3})	ρ (kg m^{-3})	μ (mPa s)	Ar	Re_t	n	a	b
1	Copper	610	8800	1000	1.307	10167.21	115.07	2.74	0.7602	0.2449
2	Copper	1300	8800	1000	1.307	98410.74	479.10	2.44	0.6717	0.3485
3	Copper	1060	8800	1000	1.307	53349.46	328.20	2.50	0.7093	0.3107
4	Copper	700	8800	1000	1.307	15364.08	150.02	2.67	0.7528	0.2559
5	Copper	886	8800	997	7.700	31153.98	234.65	2.57	0.7311	0.2832
6	Copper	1200	8800	997	7.700	77402.71	413.25	2.46	0.6876	0.3321
7	TiO_2	516	5600	988	0.550	20302.01	179.14	2.63	0.7451	0.2674
8	TiO_2	666	5600	988	0.550	43652.85	289.73	2.52	0.7199	0.2941
9	TiO_2	816	5600	988	0.550	80289.79	422.66	2.46	0.6871	0.3333
10	TiO_2	816	5600	997	0.890	30881.30	233.36	2.57	0.7333	0.2821
11	TiO_2	896	5600	997	0.890	40883.60	278.13	2.53	0.7216	0.2952
12	TiO_2	1019	5600	997	0.890	60137.82	353.54	2.49	0.7046	0.3154
13	Iron	1030	7300	997	0.890	85043.89	437.92	2.45	0.6824	0.3393
14	Iron	966	7300	997	0.890	70155.68	388.89	2.47	0.6951	0.3247
15	Iron	942	7300	997	0.890	65055.52	371.17	2.48	0.7003	0.3201
16	Lead glass	1266	2900	997	0.890	47678.66	306.09	2.51	0.7150	0.3038
17	Lead glass	1166	2900	997	0.890	37249.34	262.43	2.54	0.7241	0.2913
18	Lead glass	999	2900	997	0.890	23427.15	196.11	2.61	0.7424	0.2700
19	Glass beads	999	2450	997	0.890	17887.36	165.29	2.65	0.7496	0.2618
20	Glass beads	1224	2450	998	1.002	26995.77	214.44	2.59	0.7365	0.2771
21	ZrO_2	276	5600	988	0.550	3106.83	52.59	2.98	0.7729	0.2243
22	Copper	1466	8800	997	7.700	4055.52	62.91	2.92	0.7711	0.2273
23	Zirconia	700	3800	1000	1.307	5515.00	77.17	2.86	0.7692	0.2316
24	Copper	1666	8800	997	7.700	5952.09	81.15	2.84	0.7683	0.2329
25	Glass beads	800	2450	998	1.002	7249.38	92.37	2.80	0.7664	0.2370
26	Copper	460	8800	997	0.890	9378.19	109.22	2.76	0.7620	0.2422
27	Copper	600	8800	1000	1.307	9675.34	111.45	2.75	0.7622	0.2438
28	Glass beads	891	2450	998	1.002	10015.30	113.97	2.74	0.7610	0.2448
29	Glass beads	1700	2450	997	0.890	88144.77	447.68	2.45	0.6798	0.3405
30	Glass beads	1760	2450	997	0.890	97811.02	477.29	2.44	0.6732	0.3483
31	Glass beads	707	2450	998	1.002	5003.67	72.36	2.88	0.7697	0.2302
32	Glass beads	565	2450	998	1.002	3997.07	62.31	2.92	0.7718	0.2268
33	Copper	574	8800	998	1.002	14388.18	143.86	2.68	0.7532	0.2553
34	ZrO_2	229	8800	988	0.550	3005.86	51.42	2.98	0.7728	0.2240
35	Glass beads	1730	2450	997	0.890	92894.09	462.38	2.44	0.6770	0.3460

$$a = 0.7880 - 0.00009Ar^{0.625},$$
$$50 < Re_t < 500, \quad 3000 < Ar < 10^5 \tag{23}$$

When the bed voidages calculated from the Richardson–Zaki equation at different velocities are substituted into Eq. (21), the apparent drag force should balance with the effective gravitational force, i.e. the ratio of the LHS and RHS of Eq. (21) should be equal to 1. Fig. 7 shows a verification for the revised Eq. (21). In the same figure a comparison

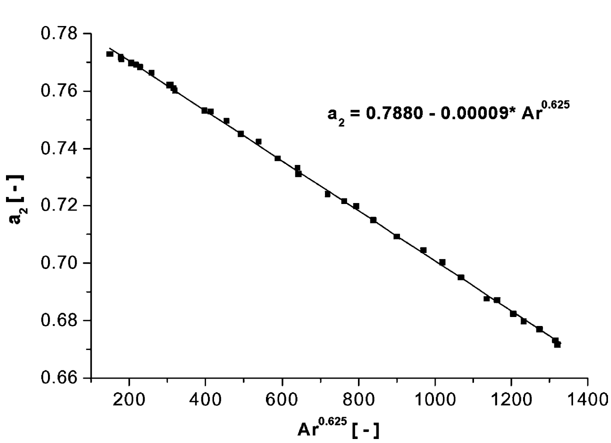

Fig. 5. Dependence of a_2 in Eq. (17) on Archimedes number.

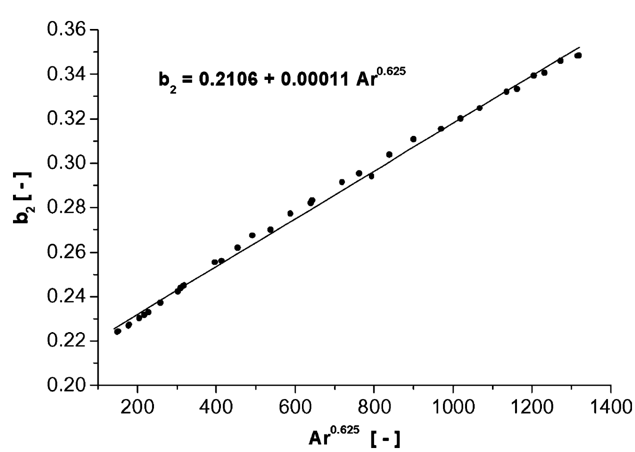

Fig. 6. Dependence of b_2 in Eq. (17) on Archimedes number.

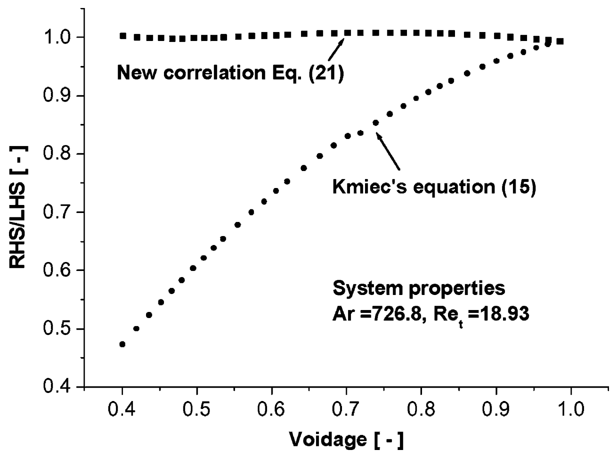

Fig. 7. Verification of new correlation Eq. (21) and comparison with Kmiec's Eq. (15).

between the revised Eq. (21) and Kmiec's Eq. (15) is given. As can be clearly seen, the ratio of the LHS and the RHS of Eq. (21) is equal to 1 for a whole range of voidage. This implies that a more accurate fluid-particles interaction correlation has been established. Correlation (21) is developed for conventional liquid–solid fluidization systems, in which the particles are in a uniformly suspended state, the liquid velocity and bed voidage distributions are homogeneous in both the axial and the radial directions. Therefore, it may not be applicable for circulating fluidization due to the non-uniform flow structure [34].

5. Conclusions

On the basis of the Richardson–Zaki equation, a new general liquid-particle interaction correlation Eq. (21) is developed under intermediate regime. The constant a in the correlation is a function of Archimedes number which offers a degree of flexibility for different systems. This correlation provides a good description on the apparent drag force F_d and the effective gravity force F_g in fluidized beds. Its accuracy and applicability in a wide range make it very useful for prediction of the expansion characteristic.

In comparison with the Richardson–Zaki equation, the practical advantage of proposed Eq. (21) is that the definitions of F_g and F_d have a clear physical meaning. System properties and operating conditions are considered, which allow further investigation of the hydrodynamics for more complex systems, such as to predict segregation phenomena and mixing composition in binary-solid fluidized beds [35].

In a fundamental sense, with addition of newly developed Eq. (21), equivalence correlations between equilibrium forces and the Richardson–Zaki equation can be found for laminar Eq. (10), intermediate Eq. (21), and turbulent Eq. (15) regimes.

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